[2] $\boxed{(1+y^2)u\frac{du}{dy} = 2yu^2}$ WHERE $u = \frac{dy}{dx}$ ½

½ $\boxed{\int \frac{1}{u}\,du = \int \frac{2y}{1+y^2}\,dy}$ $\longrightarrow$

½ $v = 1+y^2$

$\int \frac{1}{v}\,dv$

$\boxed{\ln|u| = \ln(1+y^2) + C}$

½ $\boxed{\frac{dy}{dx} = u = C(1+y^2)}$

½ $\boxed{\int \frac{1}{1+y^2}\,dy = \int C\,dx}$

½ $\boxed{\tan^{-1} y = Cx + K}$

½ $\boxed{y = \tan(Cx+K)}$

IS $u = 0$ A SOL'N?

$\frac{dy}{dx} = 0 \rightarrow y = C \rightarrow y' = y'' = 0$

$(1+C^2)0 = 2C(0)^2$ ✓

NOT SINGULAR, SINCE

$y = \tan(0x+K) = \tan K$

IS A CONSTANT SOL'N IN THE FAMILY

½

[3] $\underline{r^2+6r+9=0 \to r=-3,-3} \to \underline{y_1 = x^{-3},\ y_2 = x^{-3}\ln x}$ $\frac{1}{2}$ $\frac{1}{2}$

$$g = \frac{\frac{1}{x^3(\ln x)^2}}{x^2} = \boxed{\frac{1}{x^5(\ln x)^2}} \ \tfrac{1}{2} \qquad \boxed{W = \begin{vmatrix} x^{-3} & x^{-3}\ln x \\ -3x^{-4} & -3x^{-4}\ln x + x^{-4} \end{vmatrix} = x^{-7}}$$

$$y_p = -x^{-3}\int \frac{\frac{1}{x^5(\ln x)^2}x^{-3}\ln x}{x^{-7}}\,dx + x^{-3}\ln x \int \frac{\frac{1}{x^5(\ln x)^2}\,x^{-3}}{x^{-7}}\,dx \quad \tfrac{1}{2}$$

$$\tfrac{1}{2} \quad = \boxed{-x^{-3}\int \frac{1}{x\ln x}\,dx} + \boxed{x^{-3}\ln x \int \frac{1}{x(\ln x)^2}\,dx} \ \tfrac{1}{2}$$

$u = \ln x$ $\qquad$ $u=\ln x$

$\frac{1}{2}$ $\int \frac{1}{u}\,du$ $\qquad$ $\frac{1}{2}$ $\int \frac{1}{u^2}\,du$

$$= \boxed{-x^{-3}\ln|\ln x|} + \boxed{x^{-3}\ln x\left(-\frac{1}{\ln x}\right)}$$

$$= -x^{-3}\ln|\ln x| \underbrace{- x^{-3}}_{\text{ALREADY IN } y_h}$$

$$y = \overset{\frac{1}{2}}{\boxed{-x^{-3}\ln|\ln x|}} + \boxed{c_1 x^{-3} + c_2 x^{-3}\ln x} \ \tfrac{1}{2}$$

[4]

$\frac{1}{2}$

$$\boxed{\begin{aligned}(3D-1)[x]+(2D-9)[y]&=e^{2t} \quad ①\\ D[x]+(D-4)[y]&=4 \quad ②\end{aligned}}$$

$$\begin{array}{ll} -D[①] & -D(3D-1)[x]-D(2D-9)[y]=-D[e^{2t}]=-2e^{2t} \\ (3D-1)[②] & D(3D-1)[x]+(D-4)(3D-1)[y]=(3D-1)[4]=-4 \end{array} \quad \frac{1}{2}$$

$$(-2D^2+9D+3D^2-13D+4)[y]=-2e^{2t}-4$$

$$y''-4y'+4y=\boxed{(D^2-4D+4)[y]=-2e^{2t}-4} \quad \frac{1}{2}$$

$$\boxed{\begin{aligned}&r^2-4r+4=0 \longrightarrow r=2,2\\ &y_h=c_1e^{2t}+c_2te^{2t}\end{aligned}} \quad \frac{1}{2}$$

$$\boxed{y_p=At^2e^{2t}+B} \quad \frac{1}{2}$$

$$y_p'=2At^2e^{2t}+2Ate^{2t}$$

$$y_p''=4At^2e^{2t}+4Ate^{2t}+4Ate^{2t}+2Ae^{2t}$$

$$\begin{array}{rl} y_p'' = & 4At^2e^{2t}+8Ate^{2t}+2Ae^{2t} \\ -4y_p' & -8At^2e^{2t}-8Ate^{2t} \quad\quad 1 \\ +4y_p & +4At^2e^{2t} \quad\quad\quad\quad +4B \end{array}$$

$$=\boxed{2Ae^{2t}+4B=-2e^{2t}-4} \quad \frac{1}{2}$$

$$2A=-2 \qquad 4B=-4$$

$$A=-1 \qquad B=-1$$

$$y=\underbrace{\boxed{-t^2e^{2t}-1}}_{\frac{1}{2}}+\underbrace{\boxed{c_1e^{2t}+c_2te^{2t}}}_{\frac{1}{2}}$$

$$\begin{array}{ll} (D-4)[①] & (D-4)(3D-1)[x]+(D-4)(2D-9)[y]=(D-4)[e^{2t}] \\ & \qquad =2e^{2t}-4e^{2t}=-2e^{2t} \\ -(2D-9)[②] & -D(2D-9)[x]-(D-4)(2D-9)[y]=-(2D-9)[4] \\ & \qquad =36 \end{array} \quad \frac{1}{2}$$

$$\frac{1}{2} \quad \boxed{(D^2-4D+4)[x]=-2e^{2t}+36}$$

$$x_h=k_1e^{2t}+k_2te^{2t}$$

$$x_p=Ct^2e^{2t}+E$$

$$x_p''-4x_p'+4x_p=\boxed{2Ce^{2t}+4E=-2e^{2t}+36} \quad \frac{1}{2}$$

$$2C=-2 \qquad 4E=36$$

$$C=-1 \qquad E=9$$

$$x=\underbrace{\boxed{-t^2e^{2t}+9}}_{\frac{1}{2}}+\underbrace{\boxed{k_1e^{2t}+k_2te^{2t}}}_{\frac{1}{2}}$$

$$
\begin{aligned}
x' &= \boxed{\begin{aligned}&\cancel{-2t^2e^{2t}} - 2te^{2t} && + 2k_1e^{2t} + k_2e^{2t} + 2k_2te^{2t} \\ +y' \quad &\cancel{-2t^2e^{2t}} - 2te^{2t} && + 2c_1e^{2t} + c_2e^{2t} + 2c_2te^{2t} \\ -4y \quad &\cancel{+4t^2e^{2t}} \qquad\qquad + 4 && - 4c_1e^{2t} \qquad\qquad - 4c_2te^{2t}\end{aligned}} \quad 1 \\
&= -4te^{2t} + 4 + (2k_1 + k_2 - 2c_1 + c_2)e^{2t} + (2k_2 - 2c_2)te^{2t} \\
&= \boxed{4 + (2k_1 + k_2 - 2c_1 + c_2)e^{2t} + (2k_2 - 2c_2 - 4)te^{2t}} \quad 1
\end{aligned}
$$

$$\boxed{2k_1 + k_2 - 2c_1 + c_2 = 0 \qquad 2k_2 - 2c_2 - 4 = 0} \quad \tfrac{1}{2}$$

$$2k_1 + c_2 + 2 - 2c_1 + c_2 = 0 \qquad k_2 = c_2 + 2$$

$$2k_1 - 2c_1 + 2c_2 + 2 = 0 \qquad \text{OR } c_2 = k_2 - 2$$

$$k_1 = c_1 - c_2 - 1 \qquad 2k_1 + k_2 - 2c_1 + k_2 - 2 = 0$$

$$2k_1 + 2k_2 - 2c_1 - 2 = 0$$

$$c_1 = k_1 + k_2 - 1$$

$$\boxed{\begin{aligned} x &= -t^2e^{2t} + 9 + k_1e^{2t} + k_2te^{2t} \\ y &= -t^2e^{2t} - 1 + (k_1 + k_2 - 1)e^{2t} + (k_2 - 2)te^{2t} \end{aligned}}$$

OR

$$\boxed{\begin{aligned} x &= -t^2e^{2t} + 9 + (c_1 - c_2 - 1)e^{2t} + (c_2 + 2)te^{2t} \\ y &= -t^2e^{2t} - 1 + c_1e^{2t} + c_2te^{2t} \end{aligned}}$$

EITHER SET IS OK

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